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Quantized De Sitter Space, the Connection to the Pauli Principle and an Application to Feynman's Relativistic Quark Theory II

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Abstract

A continuation of a previous paper, in which a model of a quantized space-time theory has been investigated, considers further problems of a quantized De Sitter space. There will be shown that a De Sitter space is a very useful starting point to a non-local relativistic quantum field theory, containing the Pauli principle, for the theory of elementary particles, as a connection to Feynman's relativistic quark theory, where the group $SU(3)$ has a particular importance, will be discussed. This method offers the possibility of treating weak local differences from a space with De Sitter metric as a perturbation. Therefore the problem of a fundamental elementary length l_0 must be considered in connection with the general theory of relativity.

1. *Introduction*

In a previous paper (Ulmer, 1973) a connection between time definition in the theory of relativity and the Pauli principle has been stated. The corresponding mathematical formulation of this connection has involved the consideration of a fundamental length l_0 and, as a consequence, a quantized De Sitter space, where the curvature can only assume discrete values. Now some properties of a field equation for a particle, which represents a source of a field and which is interacting with its own field, seem to be necessary in this paper. For this purpose, we need some repetitions of the principles and problems of relativistic quantum theory. By means of the very well-known field equation

$$\Box \Psi_\mu = \frac{m^2 c^2}{\hbar^2} \Psi_\mu \tag{1.1}$$

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where $\Box = \nabla^2 - 1/c^2.(\partial^2/\partial t^2)$, Pauli (1941) derived the result that Fermi particles (odd spin) have to be quantized in consistence with

$$\Psi_\mu(x)\,\Psi_\lambda{}^+(x') + \Psi_\lambda{}^+(x')\,\Psi_\mu(x) = \delta_{\mu\lambda}\,\delta(x - x') \tag{1.2}$$

and Bose particles in consistence with

$$\phi(x)\,\phi^+(x') - \phi^+(x')\,\phi(x) = \delta(x - x') \tag{1.3}$$

In the latter case we understand particles with integral spin. Equation (1.2) is the result of the postulate that the energy of Fermi particles shall remain positive definite. Therefore Pauli considered the very general case of a situation which had been given first by Dirac's relativistic wave equation

$$\gamma^\nu \frac{\partial \Psi_\mu}{\partial x_\nu} = \frac{mc}{\hbar}\Psi_\mu$$
$$\gamma^\nu \gamma^\lambda + \gamma^\lambda \gamma^\nu = 2\delta^{\nu\lambda} \tag{1.4}$$

Without taking account of the exclusion principle, the interpretation of (1.4) has profound difficulties concerning the stability of an electron. This particle would have to shed its positive energy in a very rapid time, by emitting electromagnetic waves. An equilibrium between radiation and self-energy of the particle would never be possible, and Dirac solved this problem with the help of the hole theory, where the assumption of the Pauli principle is absolutely necessary. Because of the profound connection between energy, matter and field in the theory of relativity, the positive definite energy of a Fermi particle, which has to obey the Pauli principle, is equivalent to the fundamental problem of the stability of matter. Therefore the stability of matter has to be assumed in equations (1.1)–(1.4), because the local quantum field theory, founded by these equations, describes a particle without any structure. (In reality, the problems of the infinities, connected with local quantum field theories, exist already in non-relativistic quantum mechanics and classical electrodynamics. But the constant number of particles is the cause of those difficulties of no concern here.)

In the previous paper (Ulmer, 1973), we concluded that the relativistic definition of time and its application to the uncertainty relation directs to the statement: Any information about particles, which concerns the quantum mechanical behaviour of them at the same time, cannot be given, because a determination of the state vectors $\psi_1(t_1)$, $\psi_1'(t_1')$ involves the application of the Lorentz transformation

$$x_\mu' = L_\mu{}^\nu x_\nu + a_\mu \tag{1.5}$$

(We use the definition of $L_\mu{}^\nu$, where this transformation (Schmutzer, 1968) depends on three speed components.) It is impossible to define the rest system $\Sigma'(x_\nu')$ in which the use of a rest mass m for a material particle can only be justified, and the observation system $\Sigma(x_\nu)$, because the validity of the uncertainty relation does not permit the definition of a relationship between $\Sigma(x_\nu)$ and $\Sigma'(x'_\nu)$. The scalar product of the four vectors in special relativity

$$p^\mu p_\mu = (p^\mu)'(p_\mu)'$$

or

$$E^2 = p^2 c^2 + m_0{}^2 c^4 \tag{1.6}$$

where $(p^\mu)'(p_\mu)' = m_0{}^2 c^2$ is connected with the rest system $\Sigma'(x'_\nu)$, is together with the postulate $xp - px = i\hbar$ the usual starting point of the relativistic quantum theory. In classical special relativity equation (1.6) can always be defined, as we have no restrictions by the uncertainty relation. But this uncertainty postulate causes difficult problems in the context of the definition of the rest system and rest mass and application of (1.5) and (1.6). Therefore the relativistic quantum field theory is not proper for the calculation of the masses of elementary particles and for the stability of matter without the additional consistency postulate of positive definite energies or the Pauli principle.

In a relativistic theory, there is no exceptional position of time coordinates and the above conclusion is compatible with the exclusion principle. In this context it should be emphasized that Feynman (1948) and Stückelberg (1938) succeeded in interpreting Dirac's hole theory, where the Pauli principle has to be assumed, in terms of electrons, moving forwards in time, and positrons, moving backwards in time. This interpretation of the hole theory has already shown the relationship between the definition of time and the Pauli principle.

2. *De Sitter Space in a Non-local Quantum Field Theory*

After these qualitative comments, we continue our investigations, based on the previous paper (Ulmer, 1973). We should mention the very important fact that in classical special (or general) theory of relativity a 'many-body-system' involves in the same way a 'many-time-system', but the corresponding transformation laws always exist, thus we are able to state (or predict) the particular time difference in each system. A system of microparticles does not permit, as a consequence of the uncertainty relation, the application of those transformation laws in an unlimited way. Therefore transformation (1.6) must contain statistical information, and there are some reasons that the original homogeneous Lorentz group cannot be maintained in subatomic structures, as it is also not possible to make use of the Lorentz group without modifications at very far distances (cosmology). In both cases the classical Minkowski space, on which the local quantum field theory is based, must be abandoned (Weyl, 1923; Bopp, 1967).

Commutator rules of the kind

$$\phi(t)\, t' - t'\, \phi(t) \neq O \tag{2.1a}$$

or, in more general form,

$$\phi(x_\nu)\, x'_\nu - x'_\nu\, \phi(x_\nu) \neq O \tag{2.1b}$$

have been discussed recently by some authors (e.g., a report given by Kirzhnits (1971)). In the context of our considerations to the Pauli principle (previous paper (Ulmer, 1973)) commutator relations like (2.1a) seem to be justified and necessary, because this inequality (2.1a) prohibits information about elementary events at the same time. But postulate (2.1a) would involve an exceptional position of time coordinates, and the principles of relativity would not be satisfied in an acceptable way. Therefore the commutation rules (2.1b) form a more adequate starting point. The introduction of the commutator relations (2.1b) can be understood as a model of a non-local quantum field theory, but from an axiomatic point of view this inequality is not satisfactory, because we must investigate those algebraic foundations from which inequality (2.1b) can be specified in more detail. A more precise connection with the field equations (1.1)–(1.4) seems to be necessary. In the local quantum field theory x, x', and t represent only parameters of a field operator $\Phi(x,t)$, satisfying the algebraic relations: For $\varepsilon > \theta$ there exists $\|x - x'\| < \varepsilon$ such that

$$[\phi(x), \phi^+(x')]_\pm = \delta(\varepsilon) \tag{2.2}$$

where $\|x - x'\| < \varepsilon$ is space-like and $\delta(\varepsilon)$ the δ-distribution. From (2.1b) we can conclude a generalization of (2.2), because (2.1b) permits with the same justification the postulate of commutator (or anticommutator) rules of the form

$$[\phi(x_\nu), \phi^+(x'_\nu)]_\pm \neq O, \quad \|x_\nu - x'_\nu\| \neq O \tag{2.3}$$

In the cases of (2.1) and (2.3) x_ν and x'_ν represent operators themselves, which cannot commute with each other. Therefore commutator rules of the kind

$$\begin{aligned} x_\nu x'_\mu - x'_\mu x_\nu &= l_0^2 \gamma_{\nu\mu} \\ x^\nu x^{\mu'} - x^{\mu'} x^\nu &= l_0^2 \gamma^{\nu\mu} \\ x^\nu \gamma_\mu x^{\mu'} - \gamma_\mu x^{\mu'} x^\nu &= l_0^2 \gamma^\nu \end{aligned} \tag{2.4}$$

have been proposed (Ulmer, 1973), where the introduced matrix $\gamma_{\nu\mu}$ has to satisfy the principles of special (or general) relativity in the following way: $\Sigma'(x'_\mu)$ means the system, where the particle rests, and $\Sigma(x_\mu)$ represents those coordinates, measured by an apparatus Σ. Any other measurement apparatus shall observe $\tilde{\Sigma}(x_\mu)$, and the transformation laws between the classical measurement apparatus Σ and $\tilde{\Sigma}$ have to be given according to the principles of special (or general) relativity. A very important application of (2.4) seems to be the inhomogeneous Lorentz transformation (Poincaré group). This case leads to a quantized De Sitter space, whilst a restriction to the homogeneous proper Lorentz transformation has no meaning, and we have to discuss the following problem:

$$\tilde{\gamma} = U(\tilde{\Sigma})\gamma, \qquad \gamma = \gamma^+, \qquad x = x^+, \qquad x' = x'^+ \tag{2.5}$$

where $U(\tilde{\Sigma})$ is an unitary transformation. L (Schmutzer, 1968) is given by

$$x'_\mu = L_\mu{}^\nu x_\nu + a_\mu$$

$$L_\mu{}^\nu = \begin{pmatrix} \left(((1 - v^2/c^2)^{-1/2} - 1)\dfrac{v_i v_j}{v^2} + \delta_j{}^i,\right) & L_4{}^i \\ L_j{}^4 & (1 - v^2/c^2)^{-1/2} \end{pmatrix} \quad (2.6)†$$

$$v_i = (v_1, v_2, v_3), \qquad v^2 = v^i v_i \qquad (i = 1, \ldots, 3)$$

The introduction of (2.6) permits an investigation of De Sitter groups in a spinor field:

$$-S_\mu{}^2 = x^2 + y^2 + z^2 - c^2 t^2 - a_\mu{}^+ a_\mu \tag{2.7}$$

In connection with relation (2.4) the curvature can only assume discrete values (Ulmer, 1973). From the matrix $\gamma_{\mu\nu}$ four linear combinations γ^ν (4×4 matrices) have to be formed, which have been identified with Dirac spinors (1.4). By means of differential operators, applied to (2.7) and (2.4), the following field equations have been derived:

$$-l_0{}^2 \gamma^\nu \frac{\partial \phi_\phi}{\partial x_\nu} = \gamma^\mu (L_\mu{}^\nu x_\nu + a_\mu) \phi_\phi$$

$$-l_0{}^2 \bar{\phi}_\mu \gamma^{+\nu} \frac{\partial}{\partial x_\nu{}^+} = \bar{\phi}_\mu (L_\mu{}^\nu)^{-1} x_\nu{}^+ + a_\mu{}^+) \tag{2.8}$$

$$-S_\mu{}^2 \phi_\mu = l_0{}^4 \Box \phi_\mu$$

and

$$\bar{\phi}_\mu S_\mu{}^2 \phi_\mu = -l_0{}^4 \bar{\phi}_\mu \Box \phi_\mu \tag{2.9}$$

The application of (2.4) to (2.7) involves the additional relation

$$a_\nu{}^+ x_\nu + x_\nu a_\nu = O \tag{2.10}$$

Some properties of these equations have already been studied. At this point we should mention the interesting fact that a De Sitter space plays a very important role in general relativity. A specification to the homogeneous Lorentz group is not very interesting in high energy physics. Perhaps we should note, too, that postulate (2.10) is connected with (2.7). In general relativity, where we are able to abandon relation (2.7) for instance, if we regard a more general case, we do not need this restriction, and postulate (2.4) itself is valid for every a_ν.

In this paper we intend only to consider spinor fields in connection with very important De Sitter space, but we should refer to some publications (Schmutzer, 1964; Schrödinger, 1960; Hehl & Datta, 1971; Datta, 1971; Peres, 1962) which appeared some years ago, in which the Dirac equation and spinor fields in the general theory of relativity have been discussed in great detail. A generalization of the Dirac equation (1.4) was shown to be equivalent to a non-linear spinor equation of the Heisenberg–Pauli type, in which the non-linear term is induced by torsion.

† $L_4{}^j = -iv_j/c.(1 - v^2/c^2)^{-1/2}$.

Equations (2.8) and (2.9) represent a more general form of equations (1.1) and (1.4). The inequality (2.3) is a generalization of (2.2), because we wish to derive a formalism, where (2.2) does not vanish, in addition, for $\varepsilon \neq O$. In this case the commutator relations (2.2) cannot be reduced to the $\delta(\varepsilon)$-distribution, and instead of a δ-distribution a non-local quantum field theory must contain an expression $K_{\mu\lambda}(x_\nu, x'_\nu)$, depending on x_ν and x'_ν:

$$\phi_\mu(x_\nu)\,\phi_\lambda^{+}(x'_\nu) + \phi_\lambda^{+}(x'_\nu)\,\phi_\mu(x_\nu) = \delta_{\mu\lambda} K_{\mu\lambda} \tag{2.11}$$

It is very difficult to start with a postulate like (2.11) and to find a proper mathematical formalism. Therefore an irreducible representation of (2.11) had to be found with the help of the arguments referred to by the inequality (2.1 a, b). For this purpose the fundamental commutator rules (2.4) have been introduced as an irreducible representation of equation (2.11). Because equation (2.11) has to satisfy relativistic field operators as an additional condition with respect to x_ν and x'_ν, for example, the relativistic electrodynamics, there are some reasons for a relationship between $K_{\mu\nu}(x_\nu, x'_\nu)$ and Green's function $G(x, x')$ of an unified field of elementary particles. It is very well-known from quantum field theory that G of every linear operator L, mapping $f(x)$ into $g(x)$ with the help of

$$Lf(x) = g(x) \tag{2.12}$$

involves for the inverse problem the definition

$$f(x) = \int G(x, x')\, g(x')\, dx' \tag{2.13}$$

where L and G have to satisfy

$$LG(x, x') = \delta(x - x') \tag{2.14}$$

A specification of (2.11) for the local field theory is in the same way equivalent with a mapping of $K_{\mu\mu}$ into the δ-distribution.

We wish to illustrate the connection between (2.4) and (2.11) by means of a special example, which is easy to survey and which seems to be able to be generalized in a proper way by the use of Fourier or Laplace transformations. The operators A, B shall form a non-vanishing commutator

$$AB - BA = c \tag{2.15}$$

where c has to be constant (c-number). Now we define

$$f(A) = e^{iA}, \qquad f^{+}(B) = e^{-iB} \tag{2.16}$$

and we obtain the following expressions (Pauli, 1962):

$$\begin{aligned} e^{i(A \pm B)} &= f(A)\, f^{(+)}(B)\, e^{\pm 0 \cdot 5 \,.\, c \,.\, i \,.\, (-i)} \\ e^{i(A \pm B)} &= f^{(+)}(B)\, f(A)\, e^{\mp 0 \cdot 5 \,.\, c \,.\, i \,.\, (-i)} \end{aligned} \tag{2.17}$$

$$f(A)\, f^{+}(B) \pm f^{+}(B)\, f(A) = \lambda_1\, e^{i(A-B)} \pm \lambda_2\, e^{i(A-B)}$$

$$\lambda_1 = e^{+0 \cdot 5 \,.\, i \,.\, c \,.\, (-i)}, \qquad \lambda_2 = e^{-0 \cdot 5 \,.\, i \,.\, c \,.\, (-i)}$$

This result we can bring to a convenient form:

$$[f(A), f^{+}(B)]_{\pm} = e^{i(A-B)}(\lambda_1 \pm \lambda_2) \tag{2.18}$$

3. *Some Applications and a Connection to Feynman's Relativistic Quark Model*

According to the previous paper the solution of equation (2.9) is obtained by

$$A . \exp\{-l_2^{-0} C_\mu{}^\nu x_\nu x_\nu\} \exp\{ik^\nu x_\nu\} + A^+ \exp\{-l_0^{-2} C_\mu{}^\nu x_\nu x_\nu\} \exp\{-ik^\nu x_\nu\} \tag{3.1}$$

where $k^4 = \omega/c = \hbar\omega/\hbar c = E/\hbar c$. Equations (2.8) and (2.9) and the corresponding solution (3.1) of (2.8) permit, by means of a comparison with local relativistic quantum theory, the interpretation of a structure and self-energy for a particle interacting with its own field. Only for far distances do we obtain the usual free particle equations of relativistic quantum theory. For this case, the solutions of (3.1) are given by plane waves, coming from infinity and going to infinity, and there are no difficulties arising from the normalization volume. Considering equation (2.9), we may write this equation in another form:

$$\begin{aligned} S_\mu{}^2 \phi_\mu &= -l_0{}^4 \Box \phi_\mu \\ j_\mu &= (x^2 + y^2 + z^2 - c^2 t^2 - a_\mu{}^+ a_\mu) \phi_\mu \\ l_0{}^4 \Box \phi_\mu &= j_\mu \end{aligned} \tag{3.2}$$

With the help of (3.2) more detailed information about structure and self-interaction is possible. In analogy to electrodynamics this self-interaction of a particle with its own field can be represented by the current j_μ. It is interesting to note that this current j_μ is connected with harmonic oscillators, and the restriction to the De Sitter space involves obviously relativisitc harmonic oscillators for the description of the field of elementary particles. The particular conditions $x^2 + y^2 + z \; - c^2 t^2 = O$ (for a light beam) and a vanishing curvature ($a_\mu = 0$) lead to the second-order equation

$$\Box \phi_\mu = 0 \tag{3.2'}$$

The corresponding first-order equation

$$\gamma^\nu \frac{\partial \phi_\mu}{\partial x_\nu} = 0 \tag{3.3}$$

which represents a particle without rest mass and curvature, of which the velocity is that of light, does not satisfy P and C-invariance. In agreement with experiments equation (3.3), which is known as the Weyl equation, is used for the description of neutrinos.

A consideration of the first-order equation (2.8) shows that the neglect of the self-interaction term leads to the Dirac equation (1.4), which describes an electron, for example, without any structure, and we have to set $a_1 = a_2 = a_3 = a_4 = -a$. Now (2.8) becomes

$$\gamma^\nu \frac{\partial \phi_\mu}{\partial x_\nu} = +\frac{a}{l_0^2} \cdot \phi_\mu$$

$$a = +l_0^2 \frac{mc}{\hbar} \tag{3.4}$$

But this restriction does not agree with the solution of (2.9), which we take over from a previous paper (Ulmer, 1973), and the meaning of a_μ in (3.4) has been changed. As the Minkowski space is not considered, there are many arguments, discussed by Kirzhnits (1971), that l_0 may not be identified with $\hbar/M.c$ (M is a baryon mass).

In the theory of relativity, the total energy E (or the total mass) appears only in the fourth component of a four-vector. This fact agrees with the solution of equation (2.9). We get a_4 by:

$$a_4^{(+)} \overset{(+)}{=} \sqrt{(2)}\, l_0 \sqrt{(n - M - 1)}, \qquad n \geqslant M + 1 \tag{3.5}$$

whereas a_i ($i = 1, \ldots, 3$) is given by

$$a_i^{(+)} \overset{(+)}{=} \sqrt{(2)}\, l_0 \sqrt{(M + 1 - n)}, \qquad n \leqslant M + 1 \tag{3.6}$$

whereby $n = 0, 1, 2, \ldots$ and $M = 0, 1, 2, \ldots$ (M depends on three quantum numbers m, l, m' of the harmonic oscillator). It is necessary to note that the three a_i have to be always equal $a_1 = a_2 = a_3$ or $a_1' = a_2' = a_3'$. The group $SU(3)$ obviously plays an important role in a non-local quantum field theory, as long as we consider the De Sitter space as a sufficient foundation for a quantization. Because a_4 is connected with the total energy (or mass), the remaining a_i must also represent a mass, but this mass differs from the total mass of an elementary particle, and we may suppose that a_i has something to do with the structure. As we have solved (see Ulmer, 1973, equation (2.9)), concerning a De Sitter space, with creation and annihilation operators for each μ, we may write $x(\mu)_\nu$ in the following way:

$$x(\mu)_\nu = l_0(b_{\mu\nu}^+ + b_{\mu\nu}) \cdot \frac{1}{\sqrt{2}}, \qquad [b_{\mu\nu}^+, b_{\mu',\nu'}] = \delta_{\mu\mu} . \delta_{\nu\nu'} \tag{3.7}$$

(The index μ indicates the spinor component.)

Considerations up to now did not take account of concrete problems of modern high energy physics. Many of the present theories represent phenomenological extensions of the usual relativistic (and non-relativistic) quantum theory, for example equation (1.4). In the last few years, the application of group theory in the quark model has demonstrated many proceedings, and a symmetric, non-relativistic harmonic-oscillator quark theory has been considered for studying resonance phenomenas in high energy physics. Feynman (1970) succeeded in extending these models to a relativistic quark model, With the help of some simplifications in equations (2.9) or (3.2), we can find a very close connection to this relativistic quark model. For this reason we shall give a short report on Feynman's theory as far as the theory concerns the foundations of baryon dynamics. The application for the calculation of current matrix elements does not matter here. The non-relativistic harmonic oscillator is given by

$$E = \tfrac{1}{2}m.p^2 + \frac{m}{2}\omega_0{}^2 X^2$$

Multiplying by $2m$ and setting $m^2\omega_0{}^2 = \Omega^2 =$ constant, we shall get $2mE = p^2 + \Omega^2 X^2$. By adding a constant m^2 to the left-hand side, we obtain the squares of relativistic energies $(m^2 + E)^2$ if we neglect E^2 ($E^2 \ll m^2c^4$). Now a quark-operator for three interacting quarks can be considered:

$$K = 3\left(\sum_{j,i=1}^{3} P_i{}^2 + \frac{1}{108}\,\Omega^2[(u_i - u_j)^2]\right) + \tilde{C} \tag{3.8}$$

$\tilde{C}$ is a constant. $P_i{}^2$ represents the square of the four-vector of the momentum operator of quark i ($i = 1, \ldots, 3$): $P_i{}^2 = P_{it}P_{it} - P_{ix}P_{ix} - P_{iy}P_{iy} - P_{iz}P_{iz}$, where $P_{i\mu}$ can be replaced by $i\partial/\partial u_{i\mu}$ and $u_{i\mu}$ is the conjugate position. The propagator for baryons is given by K^{-1}. K is separable, and the external momentum $P = P_1 + P_2 + P_3$ can be separated from external motion. By means of some substitutions of (3.8) we obtain:

$$\begin{aligned}
&P_1 = \tfrac{1}{3}P - \tfrac{1}{3}\xi, \qquad P_2 = \tfrac{1}{3}P + \tfrac{1}{6}\xi - \frac{1}{2\sqrt{3}}\eta \\
&P_3 = \tfrac{1}{3}P + \tfrac{1}{6}\xi + \frac{1}{2\sqrt{3}}\eta \\
&u_1 = R - 2x, \qquad u_2 = R + x - \sqrt{(3)}\,y \\
&u_3 = R + x + \sqrt{(3)}\,y \\
&K = P^2 - \tilde{M}^2
\end{aligned} \tag{3.9}$$

hereby

$$-\tilde{M}^2 = \tfrac{1}{2}(\xi^2 + \eta^2) + \frac{\Omega^2}{2}(x^2 + y^2) + \tilde{C}$$

where $\tilde{M}^2$ was called by Feynman the mass square operator. The propagator between perturbances K^{-1} now becomes:

$$\frac{1}{P^2 - \tilde{M}^2} = \sum_i h_i(\xi,\eta)\,\frac{1}{P^2 - \tilde{M}_i{}^2}\,\bar{h}_i(\xi,\eta) \tag{3.10}$$

where (3.10) is written in terms of Gaussian eigenfunctions $h(\xi,\eta)$ of the harmonic oscillator and $\bar{h}(\xi,\eta)$ is the adjoint to h. The matrix-element of a small perturbation δK is given by

$$N_{ji} = \langle \bar{h}_j | \delta K | h_i \rangle \tag{3.11}$$

and further calculations of current matrix elements were done using creation and annihilation operators, for example:

$$x = -i(2\Omega)^{-1/2}(b_x^+ + b_x)$$
$$y = -i(2\Omega)^{-1/2}(b_y^+ + b_y), \qquad [b_\mu, b_{\mu'}^+] = \delta_{\mu\mu'} \tag{3.12}$$

It should be noted here that the author (Feynman, *et al.*, 1971) computed all matrix-elements N_{ij} with an additional simplification, because he used only space-like excited states and neglected the time variable. A comparison with the formalism in this and in the previous paper shows a very close connection to the relativistic phenomenological theory of Feynman. We wish to state here some juxtapositions:

curvature $a_i^+ a_i \Rightarrow \tilde{M}_i^2$ (supposition: $E < mc^2$)
equation (3.7) $\Rightarrow$ equation (3.12)
$P^2 - \tilde{M}^2$ $\Rightarrow$ equations (2.9) or (3.2)

Finally, some remarks on a non-local quantum field theory and the application to Feynman's relativistic quark model seem to be justified here. This close connection exists, if a De Sitter metric

$$-S_\mu^2 = x^2 + y^2 + z^2 - c^2 t^2 - a_\mu^+ . a_\mu$$

is considered. In two relevant cases this may be a specification: Very strong interactions between the particles involve a mutual influence of the curvatures. This fact has already been mentioned in the previous paper (Section 3, Electromagnetic Interactions), and Feynman therefore introduced a perturbed K' ($K' = K - \delta K$). In general relativity, a metric tensor of the form

$$g_{\mu\nu}\, dx^\mu\, dx^\nu \tag{3.13}$$

is used, whereby the restriction $g^{\nu\mu} = g_{\nu\mu} = \delta_{\nu\mu}$ is not necessary. But if we assume only an insignificant dependence of $g^{\nu\mu}(x_\nu, x_\mu)$ in the space and time variables, a treatment of $g^{\nu\mu}$ as a perturbation of a space with De Sitter metric (2.7) seems to be useful and justified. This report on a relativistic quark model, discussed by Feynman, may show that the problems of a non-local relativistic quantum field theory, based on a De Sitter space, and the connection to the Pauli principle, which we considered in this and in a previous paper, may be worth investigating, not only for the reason of better understanding of some aspects of the Pauli principle.

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